ENGINEERING BASICS

he following information helps you solve technical problems frequently encountered in designing and selecting motion control components and systems.

Torque

$$T = FR \tag{1}$$

Where:

T = Torque, lb-ft F = Force, lb R = Radius, or distance that the force is from the pivotal point, ft

Linear to rotary motion

$$N = \frac{V}{0.262D} \tag{2}$$

Where:

N = Speed of shaft rotation, rpm V = Velocity of material, fpm D = Diameter of pulley or sprocket,

in.

Horsepower

Rotating objects:

$$P = \frac{TN}{5,250} \tag{3}$$

Where: P =Power, hp T = Torque, lb-ft N = Shaft speed, rpm

Objects in linear motion:

$$P = \frac{FV}{33,000} \tag{4}$$

(5)

Where: P = Power, hp F = Force, lbV = Velocity, fpm

Pumps:

 $P = \frac{QHS}{3,960\mu}$

P =Power, hp

Q = Flow rate, gpm H = Head, ft

S =Specific gravity of fluid

 μ = Pump efficiency

Fans and blowers:

$$P = \frac{Qp}{229\mu}$$

(6)

(7)

Where:

P = Power, hp Q = Flow rate, cfm p = Pressure, psi

 $\mu = \text{Efficiency}$

Accelerating torque and force

Of rotating objects

$$T = \frac{\left(WK^2\right)\Delta N}{308t}$$

Where:

T =Torque required, lb-ft $WK^2 =$ Total inertia of load to be accelerated, lb-ft². (See Formulas 9, 10, 11, and 12.) $\Delta N =$ Change in speed, rpm t =Time to accelerate load, sec

Objects in linear motion:

$$F = \frac{W\Delta V}{1,933t} \tag{8}$$

Where: F =Force required, lb W = Weight, lb $\Delta V =$ Change in velocity, fpm t = Time to accelerate load, sec

Moment of inertia

Solid cylinder rotating about its own axis:

$$WK^2 = (1/2)WR^2$$
 (9)

Where: $WK^2 = Moment of inertia, lb-ft^2$ W = Weight of object, lbR = Radius of cylinder, ft Hollow cylinder rotating about its own axis:

$$WK^{2} = \frac{W(R_{1}^{2} + R_{2}^{2})}{2}$$
(10)

Where: $WK^2 = Moment of inertia, lb-ft^2$ W = Weight of object, lb $R_1 = Outside radius, ft$ $R_2 = Inside radius, ft$



Material in linear motion with a continuous fixed relation to a rotational speed, such as a conveyor system:

$$WK_L^2 = W \left(\frac{V}{2\pi N}\right)^2 \tag{11}$$

Where: WK_L^2 = Linear inertia, lb-ft² W = Weight of material, lb V = Linear velocity, fpm N = Rotational speed of shaft, rpm

Reflected inertia of a load through a speed reduction means — gear, chain, or belt system:

$$WK_R^2 = \frac{WK_L^2}{R_r^2} \tag{12}$$

Where: WK_R^2 = Reflected inertia, lb-ft² WK_L^2 = Load inertia, lb-ft² R_r = Reduction ratio

Duty cycle calculation

The RMS (root mean square) value of a load is one of the quantities often used to size PT components.

$$L_{RMS} = \sqrt{\frac{L_1^2 t_1 + L_2^2 t_2 + \dots + L_n^2 t_n}{t_1 + t_2 + \dots + t_n}}$$
(13)

Mechanical properties of common materials								
		Ultimate strength, psi			Modulus			
Material	Equiva- lent	Tension	Com- pression*	Shear	Yield point, ten- sion (psi)	of elasti- city, tension or com- pression (psi)	Modulus of elasti- city, shear (psi)	Weight (lb per in. ³)
Steel, forged-rolled C, 0.10-0.20 C, 0.20-0.30 C, 0.30-0.40 C, 0.60-0.80 Nickel Cast iron:	SAE 1015 SAE 1025 SAE 1035 SAE 2330	60,000 67,000 70,000 125,000 115,000	39,000 43,000 46,000 65,000	$\begin{array}{r} 48,000\\ 53,000\\ 56,000\\ 75,000\\ 92,000\end{array}$	39,000 43,000 46,000 65,000	30,000,000 30,000,000 30,000,000 30,000,00	$\begin{array}{c} 12,000,000\\ 12,000,000\\ 12,000,000\\ 12,000,000\\ 12,000,000\\ 12,000,000\end{array}$	$\begin{array}{c} 0.28 \\ 0.28 \\ 0.28 \\ 0.28 \\ 0.28 \\ 0.28 \end{array}$
Gray Gray Gray Malleable Wrought iron Stool acct:	ASTM 20 ASTM 35 ASTM 60 SAE 32510	$\begin{array}{c} 20,000\\ 35,000\\ 60,000\\ 50,000\\ 48,000\end{array}$	$\begin{array}{c} 80,000\\ 125,000\\ 145,000\\ 120,000\\ 25,000\end{array}$	$\begin{array}{c} 27,000 \\ 44,000 \\ 70,000 \\ 48,000 \\ 38,000 \end{array}$	25,000	15,000,000 20,000,000 23,000,000 27,000,000	6,000,000 	$\begin{array}{c} 0.26 \\ 0.26 \\ 0.26 \\ 0.26 \\ 0.28 \end{array}$
Low C Medium C High C Aluminum alloy:		60,000 70,000 80,000	45,000		45,000	10,000,000	3 750 000	0.28 0.28 0.28 0.10
Structural, No. 17ST Brass: Cast Annealed		10,000 58,000 40,000 54,000	35,000 35,000 18,000	35,000	35,000 35,000 18,000	10,000,000	3,750,000	0.10 0.10 0.30 0.30
Cold-drawn Bronze: Cast Cold-drawn Brick, clay Concrete 1:2:4 (28 days) Stone Timber	ASTM	96,700 22,000 85,000 	49,000 1,500 2,000 8,000 4,840	3,000 	49,000	15,000,000	6,200,000 6,000,000 1,280,000	$\begin{array}{c} 0.30\\ 0.31\\ 0.31\\ 0.72\\ 0.087\\ 0.092\\ 0.015\end{array}$

*The ultimate strength in compression for ductile materials is usually taken as the yield point. The bearing value for pins and rivets may be much higher, and for structural steel is taken as 90,000 psi.Source: S.I. Heisler, The Wiley Engineer's Desk Reference, 1984. Used with permission of John Wiley & Sons, New York.

Where:

 L_{RMS} = RMS value of the load which can be in any unit, hp, amp, etc.

- L_1 = Load during time of period 1 L_2 = Load during time of period 2, etc.
- *+* _
- t_1 = Duration of time for period 1 t_2 = Duration of time for period 1
- $t_2 =$ Duration of time for period 2,





Where: E = Modulus of elasticity,

- $lb/in.^{2}$ P = Axial load, lb
 - L = Length of object, in.
 - L = Length of object, in. A = Area of object, in.²
 - A =Area of object, in.

 Δd = Increase in length resulting from axial load, in.

General technical references

1. S.I. Heisler, *The Wiley Engineer's Desk Reference*, John S. Wiley & Sons, New York, 1984. 2. Hindehide, Zimmer-

man, *Machine Design Fundamentals*, John S. Wiley & Sons, New York, 1983.

3. K.M. Walker, Applied Mechanics for Engineering Technology, Third
(14) Edition, Reston Publishing Co. Inc.,



Metals, Metals Park, Ohio, 1983. 5. *The Smart Motion Cheat Sheet,* Amechtron Inc., Denton, Texas 1995.

CONVERSION FACTORS

Unless otherwise stated, pounds are U.S. avoirdupois, feet are U.S. standard, and seconds are mean solar.

Multiply By		To obtain	
Length			
Angstrom units	$3.937 imes10^{-9}$	in	
cm	0.3937	in.	
ft	0.30480	m	
in. (U.S.)	2.5400058	cm	
in. (British)	0.9999972	in. (U.S.)	
m	10^{10}	Angstrom units	
m	3.280833	ft	
m	39.37	in.	
m	1.09361	yd	
m	$6.2137 imes 10^{-4}$	miles (U.S. statute)	
yd	0.91440	m	
miles (U.S. statute)	5,280	ft	
Area			
cir mils	$7.854 imes10^{-7}$	in. ²	
cm^2	$1.07639 imes 10^{-3}$	$\frac{1}{\mathbf{ft}^2}$	
cm^2	0.15499969	in. ²	
ft^2	0.092903	m^2	
${ m ft}^2$	929.0341	cm^2	
in. ²	6.4516258	cm^2	
Volume			
cm ³	$3.531445 imes 10^{-5}$	ft^3	
cm ³	$2.6417 imes 10^{-4}$	gal (U.S.)	
cm^3	0.033814	oz (U.S. fluid)	
ft ³ (British)	0.9999916	ft^3	
ft^{3} (U.S.)	28.31625	L (liter)	
m^3	264.17	gal (U.S.)	
gal (British)	4,516.086	cm^3	
gal (British)	1.20094	gal (U.S.)	
gal (U.S.)	0.13368	$ft^{3}(U.S.)$	
gal (U.S.)	231	in. ³	
gal (U.S.)	3.78533	L (liter)	
gal (U.S.)	128	oz (U.S. fluid)	
oz (U.S. fluid)	29.5737	cm ³	
oz (U.S. fluid)	1.80469	in.°	
yd ³	0.76456	mຶ	
yd ³ (British)	0.76455	m ³	
Plane angle			
radian	57.29578	deg	
Weight			
Dynes	$2.24809 imes 10^{-6}$	lb	
kg	35.2740	oz (avoirdupois)	
kg	2.20462	lb	
kg	0.001	tons (metric)	
kg	0.0011023	tons (short)	
oz (avoirdupois)	28.349527	grams	
tons (long)	1,106	Ĭg	
tons (long)	2,240	lb	
tons (metric)	1,000	kg	
tons (metric)	2,204.6	lb	
tons (short)	2,000	lb	

Aultiply By		To obtain	
Velocities			
foot/gog (fog)	0 69199	mph	
meters per sec	0.00102 2 23693	mph	
rom	0.10472	radians/sec	
mph	44.7041	cm/sec	
mph	1.4667	fps	
$\label{eq:constraint} \begin{array}{l} \mbox{Temperature} \\ \mbox{deg C} = 0.555 \ (\mbox{deg F} - 32) \\ \mbox{deg F} = 1.8 \ (\mbox{deg F}) + 32 \end{array}$			
Pressure			
atmosphere	14 696	ngi	
atmosphere	10.333	kg/m ²	
lh/ft ²	4 88241	kơ/m²	
psi	70.307	grams/cm ²	
psi	703.07	kg/m ²	
Force			
Newton	0.22481	lb	
Newton	9.80	kg	
Torque			
lb-in.	0.113	(Newton-meters) N-m	
lb-ft	1.356	N-m	
lb-ft	$1.3558 imes 10^7$	dyne-cm	
oz-in.	0.00706	N-m	
Energy			
lh-in	0 113	W-Sec	
lh-in	0.113	i (joule)	
Btu	251 98	calories	
Btu	1,055.06	j	
Power			
gram-cm/sec	$9.80665 imes 10^{-5}$	W	
hp	2.545.08	 Btu (mean)/hr	
hp	550	lb-ft/sec	
hp	0.74570	kW	
hp	5,250	lb-ft/rpm	
Inertia			
Mass inertia:			
lb-in. ²	$2.93 imes10^{-4}$	kg-m ²	
oz-in. ²	$1.83 imes10^{-5}$	kg-m ²	
kg-cm ²	10^{-4}	$kg-m^2$	
Weight inertia:		~	
lb-insec ²	$1.13 imes 10^{-4}$	kg-m ²	
in-oz-sec ²	$7.06 imes10^{-3}$	kg-m ²	
lh ft soc ²	1 355	$\bar{\mathbf{k} \mathbf{q}}$ -m ²	

Source: S.I. Heisler, The Wiley Engineer's Desk Reference, 1984. Used with permission of John Wiley & Sons, New York.

The Smart Motion Cheat Sheet, Brad Grant, Amechtron Inc., Denton, Texas.

MOTION CONTROL BASICS

he first step in determining the requirements of a motion-control system is to analyze the mechanics — including friction and inertia — of the load to be positioned. Load friction can easily be determined either by estimating or by simply measuring with a torque wrench.

Inertia — the resistance of an object to accelerate or decelerate — defines the torque required to accelerate a load from one speed to another, but it excludes frictional forces. Inertia is calculated by analyzing the mechanical linkage system that is to be moved. Such systems are categorized as one of four basic drive designs: direct, gear, tangential, or leadscrew.

In the following analyses of mechanical linkage systems, the equations reflect the load parameters back to the motor shaft. A determination of what the motor "sees" is necessary for selecting both motor and its control.

Cylinder inertia

The inertia of a cylinder can be calculated based on its weight and radius, or its density, radius, and length.

Solid cylinder, Figure 1. Based on weight and radius:

$$\frac{N_{NL} - N_{FL}}{N_{FL}} \times 100$$

Based on density, radius, and length:

$$J = \frac{\pi L \rho R^4}{2\sigma}$$

Hollow cylinder, Figure 2. Based on weight and radius:

$$J = \frac{W}{2g} \left(R_o^2 + R_i^2 \right) \tag{3}$$

Based on density, radius, and length:

$$J = \frac{\pi L \rho}{2g} \left(R_o^4 - R_i^4 \right) \tag{4}$$

With these equations, the inertia of mechanical components (such as shafts, gears, drive rollers) can be calculated. Then, the load inertia and friction are reflected through the mechanical linkage system to determine



Figure 1 — Solid cylinder.



Figure 2 — Hollow cylinder.



Figure 3 — Direct drive. Load is coupled directly to motor without any speed changing device.

motor requirements.

Example: If a cylinder is a leadscrew with a radius of 0.312 in. and a length of 22 in., the inertia can be calculated by using Table 1 and substituting in equation 2:

$$J = \frac{\pi L \rho R^4}{2g} = \frac{\pi (22)(0.28)(0.312)^4}{2(386)}$$
$$= 0.000237 \text{ lb-in.-sec}^2$$

. . .

Direct drive

(2)

The simplest drive system is a direct drive, Figure 3. Because there are no mechanical linkages involved. The load parameters are directly transmitted to the motor. The speed of the motor is the same as that of the load, so the load friction is the friction the motor must overcome, and load inertia is what the motor "sees." Therefore, the total inertia is the load inertia plus the motor inertia.

$$J_t = J_l + J_m$$

(5)

Nomenclature:

	rad/sec ²
<i>e</i> =	Efficiency
$F_{i} =$	Load force. lb
$F_{c} =$	Friction force lb
\vec{F}_{a}	Preload force lb
1 pf -	Crowitational constant
g =	$\frac{1}{2}$
-	386 in./sec ²
$I_{acc} =$	Current during
	acceleration, A
$I_{rms} =$	Root-mean-squared
	current, A
<i>J</i> =	Inertia, lb-insec ²
$J_{ls} =$	Leadscrew inertia.
- 13	lb-in -sec ²
. <i>T.</i> –	Load inertia lh-in -sec ²
	Motor in ortical h in aaa^2
$J_m =$	motor mertia, ib-msec
$J_t =$	Total inertia, lb-insec
$J_p =$	Pulley inertia, lb-insec ²
$K_t =$	Torque constant, lb-in./A
L =	Length, in.
$\mu =$	Coefficient of friction
N =	Gear ratio
$N_{r} =$	Number of load gear teeth
N_{-}	Number of motor goar
1 V m-	tooth
<i>p</i> =	Density, Ib/in."
P=	Pitch, rev/in.
P_{del} =	Power delivered to the
	load, W
$P_{diss} =$	Power (heat) dissipated
	by the motor, W
$P_p =$	Total power, W
R =	Radius in
R	Inner radius in
$n_i - D$	milei raurus, m.
$n_m =$	Motor registeres 0
D	Motor resistance, Ω
$R_o =$	Motor resistance, Ω Outer radius, in.
$R_o = S_l =$	Motor resistance, Ω Outer radius, in. Load speed, rpm
$R_o = S_l = S_m =$	Motor resistance, Ω Outer radius, in. Load speed, rpm Motor speed, rpm
$R_o =$ $S_l =$ $S_m =$ $t_{acc} =$	Motor resistance, Ω Outer radius, in. Load speed, rpm Motor speed, rpm Acceleration time, sec
$R_o =$ $S_l =$ $S_m =$ $t_{acc} =$ $t_{dec} =$	Motor resistance, Ω Outer radius, in. Load speed, rpm Motor speed, rpm Acceleration time, sec Deceleration time, sec
$R_o =$ $S_l =$ $S_m =$ $t_{acc} =$ $t_{dec} =$ $t_{idle} =$	Motor resistance, Ω Outer radius, in. Load speed, rpm Motor speed, rpm Acceleration time, sec Deceleration time, sec Idle time, sec
$R_o =$ $S_l =$ $S_m =$ $t_{acc} =$ $t_{dec} =$ $t_{idle} =$ $t_{run} =$	Motor resistance, Ω Outer radius, in. Load speed, rpm Motor speed, rpm Acceleration time, sec Deceleration time, sec Idle time, sec Run time, sec
$R_o =$ $S_l =$ $S_m =$ $t_{acc} =$ $t_{dec} =$ $t_{idle} =$ $t_{run} =$ T =	 Motor resistance, Ω Outer radius, in. Load speed, rpm Motor speed, rpm Acceleration time, sec Deceleration time, sec Idle time, sec Run time, sec Torque, lb-in.
$R_o =$ $S_l =$ $S_m =$ $t_{acc} =$ $t_{dec} =$ $t_{idle} =$ $t_{run} =$ T = $T_{acc} =$	 Motor resistance, Ω Outer radius, in. Load speed, rpm Motor speed, rpm Acceleration time, sec Deceleration time, sec Idle time, sec Run time, sec Torque, lb-in. Acceleration torque, lb-in.
$R_o =$ $S_l =$ $S_m =$ $t_{acc} =$ $t_{dec} =$ $t_{run} =$ T = $T_{acc} =$ $T_{acc} =$ $T_{dec} =$	 Motor resistance, Ω Outer radius, in. Load speed, rpm Motor speed, rpm Acceleration time, sec Deceleration time, sec Idle time, sec Run time, sec Torque, lb-in. Acceleration torque, lb-in. Deceleration torque, lb-in.
$ \begin{array}{c} R_{o}=\\ S_{l}=\\ S_{m}=\\ t_{acc}=\\ t_{dec}=\\ t_{idle}=\\ t_{run}=\\ T=\\ T_{acc}=\\ T_{dec}=\\ T_{f}=\\ \end{array} $	 Motor resistance, Ω Outer radius, in. Load speed, rpm Motor speed, rpm Acceleration time, sec Deceleration time, sec Idle time, sec Run time, sec Torque, lb-in. Acceleration torque, lb-in. Deceleration torque, lb-in. Friction torque, lb-in.
$ \begin{array}{c} R_o = \\ S_l = \\ S_m = \\ t_{acc} = \\ t_{dec} = \\ t_{idle} = \\ t_{run} = \\ T = \\ T_{acc} = \\ T_{dec} = \\ T_f = \\ T_l = \end{array} $	 Motor resistance, Ω Outer radius, in. Load speed, rpm Motor speed, rpm Acceleration time, sec Deceleration time, sec Idle time, sec Run time, sec Torque, lb-in. Acceleration torque, lb-in. Friction torque, lb-in. Load torque, lb-in.
$ \begin{array}{c} R_o = \\ S_l = \\ S_m = \\ t_{acc} = \\ t_{dec} = \\ t_{idle} = \\ t_{run} = \\ T = \\ T_{acc} = \\ T_{dec} = \\ T_f = \\ T_l = \\ T_m = \end{array} $	 Motor resistance, Ω Outer radius, in. Load speed, rpm Motor speed, rpm Acceleration time, sec Deceleration time, sec Idle time, sec Run time, sec Torque, lb-in. Acceleration torque, lb-in. Friction torque, lb-in. Load torque, lb-in. Motor torque, lb-in.
$ \begin{array}{c} R_o = \\ S_l = \\ S_m = \\ t_{acc} = \\ t_{dec} = \\ t_{idle} = \\ t_{run} = \\ T_{acc} = \\ T_{dec} = \\ T_{dec} = \\ T_l = \\ T_l = \\ T_r = \\ T_r = \end{array} $	 Motor resistance, Ω Outer radius, in. Load speed, rpm Motor speed, rpm Acceleration time, sec Deceleration time, sec Idle time, sec Run time, sec Torque, lb-in. Acceleration torque, lb-in. Friction torque, lb-in. Load torque, lb-in. Motor torque, lb-in. Torque reflected to motor,
$ \begin{array}{l} R_o = \\ S_l = \\ S_m = \\ t_{acc} = \\ t_{idle} = \\ t_{run} = \\ T_{acc} = \\ T_{acc} = \\ T_{dec} = \\ T_{l} = \\ T_l = \\ T_r = \\ T_r = \end{array} $	 Motor resistance, Ω Outer radius, in. Load speed, rpm Motor speed, rpm Acceleration time, sec Deceleration time, sec Idle time, sec Run time, sec Torque, lb-in. Acceleration torque, lb-in. Deceleration torque, lb-in. Load torque, lb-in. Motor torque, lb-in. Torque reflected to motor, lb-in.
$ \begin{array}{l} R_{o} = \\ S_{l} = \\ S_{m} = \\ t_{acc} = \\ t_{dec} = \\ t_{run} = \\ T_{acc} = \\ T_{acc} = \\ T_{dec} = \\ T_{l} = \\ T_{l} = \\ T_{r} = \\ T_{r} = \\ T_{rms} = \end{array} $	 Motor resistance, Ω Outer radius, in. Load speed, rpm Motor speed, rpm Acceleration time, sec Deceleration time, sec Idle time, sec Run time, sec Torque, lb-in. Acceleration torque, lb-in. Deceleration torque, lb-in. Load torque, lb-in. Motor torque, lb-in. Torque reflected to motor, lb-in. Root-mean-squared
$ \begin{array}{l} R_o = \\ S_l = \\ S_m = \\ t_{acc} = \\ t_{dec} = \\ t_{idle} = \\ T_{acc} = \\ T_{acc} = \\ T_{dec} = \\ T_{dec} = \\ T_l = \\ T_l = \\ T_r = \\ T_r = \\ T_{rms} = \end{array} $	 Motor resistance, Ω Outer radius, in. Load speed, rpm Motor speed, rpm Acceleration time, sec Deceleration time, sec Idle time, sec Run time, sec Torque, lb-in. Acceleration torque, lb-in. Deceleration torque, lb-in. Load torque, lb-in. Motor torque, lb-in. Torque reflected to motor, lb-in. Root-mean-squared torque, lb-in.
$ \begin{array}{l} R_{o} = \\ S_{l} = \\ S_{m} = \\ t_{acc} = \\ t_{idle} = \\ t_{idle} = \\ T_{acc} = \\ T_{acc} = \\ T_{dec} = \\ T_{dec} = \\ T_{l} = \\ T_{r} = \\ T_{r} = \\ T_{rms} = \\ T_{run} = \\ T_{run} = \end{array} $	 Motor resistance, Ω Outer radius, in. Load speed, rpm Motor speed, rpm Acceleration time, sec Deceleration time, sec Idle time, sec Run time, sec Torque, lb-in. Acceleration torque, lb-in. Deceleration torque, lb-in. Load torque, lb-in. Motor torque, lb-in. Torque reflected to motor, lb-in. Root-mean-squared torque, lb-in. Running Torque, lb-in.
$ \begin{array}{l} R_{o} = \\ S_{l} = \\ S_{m} = \\ t_{acc} = \\ t_{dec} = \\ t_{idle} = \\ T_{acc} = \\ T_{acc} = \\ T_{dec} = \\ T_{dec} = \\ T_{l} = \\ T_{r} = \\ T_{r} = \\ T_{r} = \\ T_{rms} = \\ T_{rms} = \\ T_{s} = \\ \end{array} $	 Motor resistance, Ω Outer radius, in. Load speed, rpm Motor speed, rpm Acceleration time, sec Deceleration time, sec Idle time, sec Run time, sec Torque, lb-in. Acceleration torque, lb-in. Deceleration torque, lb-in. Load torque, lb-in. Motor torque, lb-in. Torque reflected to motor, lb-in. Root-mean-squared torque, lb-in. Stall torque, lb-in.
$ \begin{array}{l} R_{o} = \\ S_{l} = \\ S_{m} = \\ t_{acc} = \\ t_{dec} = \\ t_{idle} = \\ T_{acc} = \\ T_{acc} = \\ T_{dec} = \\ T_{dec} = \\ T_{l} = \\ T_{r} = \\ T_{r} = \\ T_{r} = \\ T_{rms} = \\ T_{run} = \\ T_{s} = \\ V_{c} = \\ V_{c} = \\ \end{array} $	 Motor resistance, Ω Outer radius, in. Load speed, rpm Motor speed, rpm Acceleration time, sec Deceleration time, sec Idle time, sec Run time, sec Torque, lb-in. Acceleration torque, lb-in. Deceleration torque, lb-in. Load torque, lb-in. Motor torque, lb-in. Torque reflected to motor, lb-in. Root-mean-squared torque, lb-in. Stall torque, lb-in. Load speed ipm
$ \begin{array}{l} R_{o} = \\ S_{l} = \\ S_{m} = \\ t_{acc} = \\ t_{dec} = \\ t_{idle} = \\ T_{acc} = \\ T_{acc} = \\ T_{dec} = \\ T_{dec} = \\ T_{l} = \\ T_{r} = \\ T_{r} = \\ T_{r} = \\ T_{r} = \\ T_{rms} = \\ T_{run} = \\ T_{s} = \\ V_{l} = \\ W \end{array} $	 Motor resistance, Ω Outer radius, in. Load speed, rpm Motor speed, rpm Acceleration time, sec Deceleration time, sec Idle time, sec Run time, sec Torque, lb-in. Acceleration torque, lb-in. Deceleration torque, lb-in. Load torque, lb-in. Motor torque, lb-in. Torque reflected to motor, lb-in. Root-mean-squared torque, lb-in. Stall torque, lb-in. Load speed, ipm
$ \begin{array}{l} R_{o} = \\ R_{o} = \\ S_{l} = \\ S_{m} = \\ t_{acc} = \\ t_{acc} = \\ t_{idle} = \\ T_{rur} = \\ T_{acc} = \\ T_{dec} = \\ T_{dec} = \\ T_{l} = \\ T_{l} = \\ T_{r} = \\ T_{r} = \\ T_{r} = \\ T_{r} = \\ T_{rus} = \\ T_{s} = \\ V_{l} = \\ W \end{array} $	 Motor resistance, Ω Outer radius, in. Load speed, rpm Motor speed, rpm Acceleration time, sec Deceleration time, sec Idle time, sec Run time, sec Torque, lb-in. Acceleration torque, lb-in. Deceleration torque, lb-in. Load torque, lb-in. Motor torque, lb-in. Torque reflected to motor, lb-in. Root-mean-squared torque, lb-in. Stall torque, lb-in. Load speed, ipm Weight, lb Wairbt of head always with always

Gear drive

The mechanical linkages between the load and motor in a gear drive, Figure 4, requires reflecting the load parameters back to the motor shaft. As with any speed changing system, the load inertia reflected back to the motor is a squared function of the speed ratio.



Figure 4 — Speed changer between load and motor. Any speed changing device gearing, belt, or chain — alters the reflected inertia to the motor by the square of the speed ratio.

Motor speed:

$$S_m = S_l \times N \tag{6}$$

 \mathbf{or}

$$S_m = \frac{S_l \times N_l}{N_m}$$

Motor torque:

$$T_m = \frac{T_l}{Ne} \tag{8}$$

Reflected load inertia:

$$J_r = \frac{J_l}{N^2} \tag{9}$$

Total inertia at motor:

$$J_t = \frac{J_l}{N^2} + J_m \tag{10}$$

Example: To calculate the reflected inertia for a 6-lb, solid cylinder with a 4-in. diameter, connected through a 3:1 gear set, first use equation 1 to determine the load inertia.

$$J_{l} = \frac{WR^{2}}{2g} = \frac{6(2)^{2}}{2(386)}$$
$$= 0.031 \text{ lb} \cdot \text{in} \cdot \sec^{2}$$

To reflect this inertia through the gear set to the motor, substitute in

equation 9.

$$J_r = \frac{0.031}{3^2} = 0.0034 \text{ lb} \cdot \text{in.} \cdot \text{sec}^2$$

For high accuracy, the inertia of the gears should be included when determining total inertia. This value can be obtained from literature or calculated using the equations for the inertia of a cylinder. Gearing efficiencies should also be considered when calculating required torque values.

Tangential drive

Consisting of a timing belt and pulley, chain and sprocket, or rack and pinion, a tangential drive, Figure 5, also requires reflecting load parameters back to the motor shaft.

Motor speed:

 S_m

$$=\frac{V_l}{2\pi R}$$
 (11)

(13)

Load torque:

$$T_l = F_l R \tag{12}$$

Friction torque:

 $T_f = F_f R$

Load inertia:

(7)

$$J_l = \frac{W_{lb}R^2}{g} \tag{14}$$

Total inertia:

$$J_t = \frac{W_{lb}R^2}{g} + J_{p1} + J_{p2} + J_m \qquad (15)$$

Example: A belt and pulley arrangement will be moving a weight of 10 lb. The pulleys are hollow cylin-



Figure 5 — Tangential drive. The total load (belt plus load) is moved with a lever arm with a radius, *R*.

ders, 5-lb each, with an outer radius of 2.5 in. and an inner radius of 2.3 in.

To calculate the inertial for a hollow, cylindrical pulley, substitute in equation 3:

$$J_p = \frac{W}{2g} \left(R_o^2 + R_i^2 \right) = \frac{5}{2(386)} \left(2.5^2 + 2.3^2 \right)$$
$$= 0.0747 \text{ lb} \cdot \text{in.} - \sec^2$$

Substitute in equation 14 to determine load inertia:

$$J_l = \frac{WR^2}{g} = \frac{10(2.5)^2}{386}$$

Total inertia reflected to the motor shaft is the sum of the two pulley inertias plus the load inertia:

$$J = J_l + J_{p1} + J_{p2}$$

= 0.1619 + 0.0747 + 0.0747
= 0.3113 lb - in. - sec.²

Also, the inertia of pulleys, sprockets or pinion gears must be included to determine the total inertia.

Table 1—Material densities			
Material	Density, lb per cu in.		
Aluminum	0.096		
Copper	0.322		
Plastic	0.040		
Steel	0.280		
Wood	0.029		

Table 2—Typical leadscrew efficiencies

Туре	Efficiency
Ball-nut Acme (plastic nut) Acme (metal nut)	$0.90 \\ 0.65 \\ 0.40$

Table 3—Leadscrew coefficients of friction			
Steel on steel (dry) Steel on steel	0.58		
(lubricated)	0.15		
Teflon on steel	0.04		
Ball bushing	0.003		

Leadscrew drive

Illustrated in Figure 6, a leadscrew drive also requires reflecting the load parameters back to the motor. Both the leadscrew and the load inertia have to be considered. If a leadscrew inertia is not readily available, the equation for a cylinder may be used.



Figure 6 — Leadscrew drive.

For precision positioning, the leadscrew may be preloaded to eliminate or reduce backlash. Such preload torque can be significant and must be included, as must leadscrew efficiency.

Motor speed:

$$S_m = V_l \times P \tag{16}$$

Load torque reflected to motor:

$$T_r = \frac{1}{2\pi} \frac{F_l}{Pe} + \frac{1}{2\pi} \frac{F_{pf}}{P} \times \mu \qquad (17)$$

For typical values of leadscrew efficiency (e) and coefficient of friction (μ) , see Tables 2 and 3.

Friction force:

$$F_f = \mu \times W \tag{18}$$

(19)

Friction torque:

$$T_f = \frac{1}{2\pi} \frac{F_f}{Pe}$$

Total inertia:

$$J_t = \frac{W}{g} \left(\frac{1}{2\pi P}\right)^2 + J_{ls} + J_m \qquad (20)$$

Example: A 200-lb load is positioned by a 44-in. long leadscrew with a 0.5-in. radius and a 5-rev/in. pitch. The reflected load inertia is:

$$J_l = \frac{W}{g} \left(\frac{1}{2\pi P}\right)^2 = \frac{200}{386} \left(\frac{1}{2\pi 5}\right)^2$$
$$= 0.00052 \text{ lb} \cdot \text{in.} \cdot \sec^2$$

Leadscrew inertia is based on the

equation for inertia of a cylinder:

$$J_{ls} = \frac{\pi L \rho R^4}{2g} = \frac{\pi (44) 0.28 (0.5)^4}{2(386)}$$

= 0.00313 lb - in. - sec²

Total inertia to be connected to the motor shaft is:

$$J = J_l + J_{ls} = 0.00052 + 0.00313$$
$$= 0.00365 \text{ lb} \cdot \text{in.} \cdot \sec^2$$

Motion control system

Once the mechanics of the application have been analyzed, and the friction and inertia of the load are known, the next step is to determine the torque levels required. Then, a motor can be sized to deliver the required torque and the control sized to power the motor. If friction and inertia are not properly determined, the motion control system will either take too long to position the load, or it will be unnecessarily costly.

In a basic motion-control system, Figure 7, the load represents the mechanics being positioned. The load is coupled or connected through one of the mechanical linkages previously described.

The motor may be a traditional PMDC servo motor, a vector motor, or a brushless servo motor. Motor starting, stopping and speed are dictated by the control unit which takes a lowlevel incoming command signal and amplifies it to a higher-power level for controlling the motor.

The programmable motion controller is the brain of the motion control system and controls the motor control (amplifier). The motion controller is programmed to accomplish a specific task for a given application. This controller reads a feedback signal to monitor the position of the load. By comparing a pre-programmed,





"desired" position with the feedback position, the controller can take action to minimize an error between the actual and desired load positions.

Movement profile

A movement profile defines the desired acceleration rate, run time, speed, and deceleration rate of the load. For example, suppose with a system at rest (time =0, Figure 8), the motion controller issues a command to the motor (through the servo control) to start motion. At t=0, with full power-supply voltage and current applied, the motor has not yet started to move. At this instant, there is no feedback signal, but the error signal is large.

As friction and torque are overcome, the motor and load begin to accelerate. As the motor approaches the commanded speed, the error signal is reduced and, in turn, voltage applied to the motor is reduced. As the system stabilizes at running speed, only nominal power (voltage and current) are required to overcome friction and windage. At t=1, the load position approaches the desired position and begins to decelerate.

In applications with similar move profiles, most of the input energy is dissipated as heat. Therefore, in such systems, the motor's power dissipation capacity is the limiting factor. Thus, basic motor dynamics and power requirements must be determined to ensure adequate power capability for each motor.

Determining acceleration rate is the first step. For example, with a movement profile as shown in Figure 6, the acceleration rate can be determined from the speed and acceleration time. (Dividing the motor speed expressed in rpm by 9.55 converts the speed to radians per second).

$$\alpha_{acc} = \frac{S_m}{9.55 t_{acc}}$$
(21)
$$\alpha_{acc} = \frac{2,000}{9.55 (0.12)} = 1,745.2 \text{ rad/sec}^2$$

Acceleration torque

The torque required to accelerate the load and overcome mechanical friction is:

$$T_{acc} = J_t (\alpha_{acc}) + T_f$$
(22)
= $(J_t + J_{ls} + J_m) \alpha_{acc} + T_f$ (23)

Example: Our application, Figure 9, requires moving a load through a leadscrew. The load parameters are:

Weight of load $(W_{lb}) = 200 \text{ lb}$

Leadscrew inertia $(J_{ls}) = 0.00313$ lb-in-sec²

Friction torque $(T_f) = 0.95$ lb-in. Acceleration rate $(\alpha_{acc}) = 1745.2$ rad per sec².

Typical motor parameters are: Motor rotor inertia $(J_m) = 0.0037$ lbin.²

Continuous stall torque $(T_s) = 14.4$ lb-in.

Torque constant (K_t) = 4.8 ;b-in./A Motor resistance (R_m) = 4.5 Ω

Acceleration torque can be determined by substituting in equation 23.

$$T_{acc} = (.00052 + .00313 + .0037) 1745.2 + .95$$

= 13.75 lb - in.

Duty cycle torque

In addition to acceleration torque, the motor must be able to provide sufficient torque over the entire duty cycle or movement profile. This includes a certain amount of constant torque during the run phase, and a deceleration torque during the stopping phase.

Running torque is equal to friction torque (T_f) , in this case, 0.95 lb-in.

During the stopping phase, deceleration torque is:

$$T_{dec} = -J_t (\alpha_{acc}) + T_f$$
(24)
= -(.00052 + .00313 + .0037)1,745.2 + .95
= -11.85 lb- in.

Now, the root-mean-squared (rms) value of torque required over the movement profile can be calcuated:

$$\begin{split} T_{rms} &= \sqrt{\frac{T_{acc}^2\left(t_{acc}\right) + T_{run}^2\left(t_{run}\right) + T_{dec}^2\left(t_{dec}\right)}{t_{acc} + t_{run} + t_{dec} + t_{idle}}} \\ &= \sqrt{\frac{\left(13.75\right)^2(.12) + (.95)^2(.12) + (11.85)^2(.12)}{.12 + .12 + .12 + .3}} \\ &= 7.73 \text{ lb-in.} \end{split}$$

The motor tentatively selected for this application can supply a continuous stall torque of 14.4 lb-in., which is adequate for the application.

Control requirements

Determining a suitable control (amplifier) is the next step. The control must be able to supply sufficient acceleration current (I_{acc}), as well as continuous current (I_{rms}) for the application's duty-cycle requirements.

Required acceleration current that must be supplied to the motor is:

$$I_{acc} = \frac{T_{acc}}{K_t}$$
(25)
= $\frac{13.75}{4.8} = 2.86 \text{ A}$

Current over the duty cycle, which the control must be able to supply to the motor, is:

$$I_{rms} = \frac{T_{rms}}{K_t}$$
(26)
= $\frac{7.73}{4.8} = 1.61 \text{ A}$

Power requirements

The control must supply sufficient power for both the acceleration portion of the movement profile, as well as for the duty-cycle requirements. The two aspects of power requirements include (1) power to move the load, (P_{del}) and (2) power losses dissipated in the motor, (P_{diss}) .

Power delivered to move the load is:

$$P_{del} = \frac{T(S_m)(746)}{63,025} \tag{27}$$

Power dissipated in the motor is a function of the motor current. Thus, during acceleration, the value depends on the acceleration current (I_{acc}) ; and while running, it is a function on th rms current (I_{rms}) . Therefore, the appropriate value is used in place of "I" in the following equation.

$$P_{diss} = I^2(R_m) \tag{28}$$

The sum of (P_{del}) and (P_{diss}) determine total power requirements.

Example: Power required during the acceleration portion of the movement profile can be obtained by substituting in equations 27 and 28:

$$P_{del} = \frac{13.75(2,000)}{63,025} (746) = 325.5 \text{ W}$$
$$P_{diss} = (2.86)^2 (4.5) (1.5) = 55.2 \text{ W}$$

$$\begin{split} P_p &= P_{del} + P_{diss} \\ &= 325.5 + 55.2 = 380.7 \; \mathrm{W} \end{split}$$

Note: The factor of 1.5 in the P_{diss} calculation is a factor used to make the motor's winding resistance "hot." This is a worst-case analysis, assuming the resistance is a 155 C.

Continous power required for the duty cycle is:

$$P_{del} = \frac{7.73(2,000)}{63.025} (746) = 182.9 \text{ W}$$
$$P_{diss} = (1.61)^2 (4.5) (1.5) = 17.5 \text{ W}$$
$$P_p = 182.9 + 17.5 = 200.4 \text{ W}$$

In summary

The control selected must be capable of delivering (as a minimum) an acceleration (or peak) current of 2.86 A, and a continuous (or rms) current of 1.61 A. The power requirement calls for peak power of $380.7\ W$ and continuous power of $200.4\ W.$

To aid in selecting both motors and controls (amplifiers), many suppliers offer computer software programs to perform the iterative calculations necessary to obtain the optimum motor and control.

From articles by Baldor Electric, published in the September 1995 and March 1996 issues of PTD.

COMPUTER TERMINOLOGY

Here are some of the vocabulary used in digital communication, whether it is between user and machine, machine and machine, or user and user. They are words used by those involved with PCs, PLCs, and control devices.

BBS — Bulletin board systems offer forums, mostly local or regional. They support all types of communication, conversation, and postings.

Bit — Symbolic representation of an "On" or "Off" state of a device. In computer code, it is indicated as a "1" or a "0."

Browsers — Graphics-based software programs that let you reach a variety of locations on the Internet and move from one to the other (surf). They also retrieve and display information, both text and graphics, from these locations. Examples include Netscape and Mosaic.

Bus — Years ago, bus referred to the path or paths data traveled on the backplane of a computer board. The definition is broadening to include data traveling within the physical medium of a few wires or cables.

Byte — Eight 1s or 0s grouped together, in any combination. Each group of eight bits represents an instruction, a command, or datum.

Chat channels — Addresses on the Internet where real time "conversations" take place between groups of individuals who are signed into a particular Chat Room. Usually they have a posted subject for discussion. Many times the same individuals return to the same room at the same time each day.

Connectivity — The ability to have one device connect, attach, or communicate with another.

Data highway — Another term for bus or network. Also, a network system created by Allen-Bradley.

Distributed — In communications, a configuration where control, command execution ability, or intelligence (such as microprocessor intelligence) is spread among two or more devices.

Domain or zone — Part of an Internet address. It consists of a two or three-letter designation for the type of organization or geographical location, such as:

.comCommercial Organizations
$.govGovernment\ Departments$
$.eduEducational\ Institutions$
.milMilitary
.netNetworking Units
.orgProfessional Societies
.USUnited States
.JPJapan
.UK or .GBUnited Kingdom
.DEGermany

E-mail — Electronic mail supported across the Internet. Requires an address consisting of the Internet name or number of the recipient at the specific service provider. A CompuServe address has a series of numbers: 1111.1111@compuserve.com. No spaces are allowed between letters or numbers.

Fieldbus — A general term used to describe any bus that connects devices to microprocessor-based controls. Synonymous with devicelevel bus, sensor-actuator bus, midlevel bus.

File Transfer Protocol (FTP) — The means by which computer files and software programs are transferred from a host computer to recipient's computers.

Finger — Search capability for email addresses where location is known but e-mail address is not.

 $\label{eq:Foundation fieldbus} \textbf{Foundation fieldbus} - \textbf{A} \text{ specification for process applications}.$

Gateway — Software on a board or chip that converts one communication protocol to another. Like converting a DOS program to an Apple-based program. Sometimes gateways also convert cable types.

Gopher — Menu-based system that helps find data residing on computers at various locations. Sometimes a long process down a series of decision-tree paths.

Hub — Hardware interface device between different cable types. Connects these cables together into a network.

Integration — Sufficient communication among devices, such that performance is enhanced to a level not possible as independent devices. Devices become part of a whole as opposed to separate pieces of a system.

Internet — Worldwide network of computers and computer networks. Begun by the Defense Department in 1969 to ensure communications between colleges and universities, contractors, and government. Colleges and students have used it for a long time, but companies and individuals jumped onboard only recently.

Kbaud — A transmission rate of one byte per second.

Network — All the cabling, wiring, and software parameters and control used to connect microprocessor-based devices over long distances. Distance is less of a factor now.

Packet — Several bytes of data grouped together in a network message.

Protocol — A specification that defines input signal levels, polarities, and speeds, and a device's output signals.

Search engines — Software programs that let the user find information sites by definition, subject, or even key words, then retrieve information from these sites, including text, graphics, and sound. Examples include Yahoo, Lycos, and Alta Vista.

Usenet — A worldwide bulletin board divided into categories on which you can post news, make inquiries, or add comments.

World Wide Web — A rapidly expanding group of home pages that provide information on many subjects; individuals, companies, organizations. and governments. Usually the address looks like: http://www.foggy.com/dognews /~csmith/collars.html. There are no spaces and every letter, number, and punctuation must be exact to reach the right address. Once there, the user can usually look at sub-pages or be linked with other locations of similar interest

These definitions were taken from "PLCs bus into the future" (PTD July 1995 p. 19) and "Internet for Engineers" (PTD Aug. 1996, p. 59)